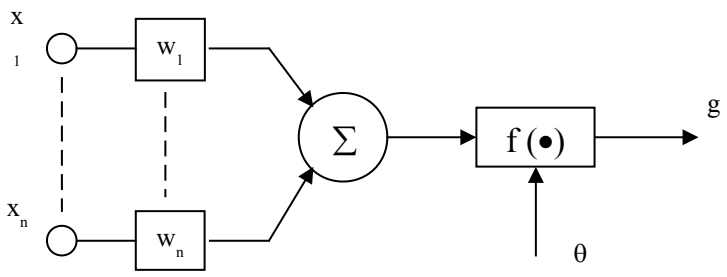
THE PERCEPTRON LEARNING RULE *(DELTA RULE)*



The perceptron model

$$g(t)=f[\sum_{i=1}^{n} x_i(t)w_i(t)-\theta(t)]$$

$w_i(t+1) = w_i(t) + \Delta w_i(t)$
$\Delta w_i(t) = \eta\, e(t)\, x_i(t)$
$e(t) = y(t) - g(t)$

where:
{x,y} are the training set
$w_i(t)$ are the perceptron weights
g(t) is the perceptron output at the time t (or step t). g(t) can be 0 or 1
e(t) is the error between the desired output y(t) and the perceptron real output g(t)
η is the learning rate with $0<\eta\leq 1$.

Example:
We want a perceptron to tell the difference between a bird and a non-bird animal.
The training set is made up by these animals: eagle, ostrich (birds) and bat, horse, fish (non-birds).

Animal	x_1	x_2	x_3	x_4	y
Eagle	1	1	1	1	1
Ostrich	0	1	1	1	1
Bat	1	0	1	1	0
Horse	0	0	0	1	0
Fish	0	1	0	1	0

Where:
x_1 is the first input which refers to the flight characteristic (1 fly, 0 don't fly)
x_2 is the second input which regards the egg-laying apparatus (1 yes, 0 no)
x_3 is the third input which refers to the question: “has it got wings?” (1 yes, 0 no)
x_4 is an imaginary input (whose value is always 1) which is used to remove the threshold θ. In this case $w_4 = -\theta$.
We assume $\eta = 1$, so $\Delta w_i(t) = \eta\, e(t)\, x_i(t) = e(t)\, x_i(t)$.

Let's start the perceptron training.

Step 0:
Initializing the weights w_1, w_2, w_3, w_4 using random values.uri
$w_1(0)=0$, $w_2(0)=1$, $w_3(0)=1$, $w_4(0)=0$

Step 1:

w_1	w_2	w_3	w_4
0	1	1	0

Showing the first animal (eagle) to the perceptron:
$x_1(1)w_1(1) + x_2(1)w_2(1) + x_3(1)w_3(1) + x_4(1)w_4(1) = 1*0 + 1*1 + 1*1 + 1*0 = 2 > 0$
So, $g(1) = 1$
$e(1) = y(1) – g(1) = 1 – 1 = 0$
In this case $\Delta w_i(1) = e(1)\, x_i(1) = 0$
The weights don't change.

Step 2:

w_1	w_2	w_3	w_4
0	1	1	0

Showing the second animal (ostrich) to the perceptron:
$x_1(2)w_1(2) + x_2(2)w_2(2) + x_3(2)w_3(2) + x_4(2)w_4(2) = 0*0 + 1*1 + 1*1 + 1*0 = 2 > 0$
So, $g(2) = 1$
$e(2) = y(2) – g(2) = 1 – 1 = 0$
In this case $\Delta w_i(2) = e(2)\, x_i = 0$
The weights don't change.

Step 3:

w_1	w_2	w_3	w_4
0	1	1	0

Showing the third animal (bat) to the perceptron:
$x_1(3)w_1(3) + x_2(3)w_2(3) + x_3(3)w_3(3) + x_4(3)w_4(3) = 1*0 + 0*1 + 1*1 + 1*0 = 1 > 0$
So, $g(3) = 1$
$e(3) = y(3) – g(3) = 0 – 1 = -1$
In this case the weights must change.
$\Delta w_1(3) = e(3)\, x_1 = -1*1 = -1$
$\Delta w_2(3) = e(3)\, x_2 = -1*0 = 0$
$\Delta w_3(3) = e(3)\, x_3 = -1*1 = -1$
$\Delta w_4(3) = e(3)\, x_4 = -1*1 = -1$
So, according to the delta rule, new weights are:
$w_1(4) = w_1(3) + \Delta w_1(3) = 0 + (-1) = -1$
$w_2(4) = w_2(3) + \Delta w_2(3) = 1 + 0 = 1$
$w_3(4) = w_3(3) + \Delta w_3(3) = 1 + (-1) = 0$
$w_4(4) = w_4(3) + \Delta w_4(3) = 0 + (-1) = -1$

Step 4:

w_1	w_2	w_3	w_4
-1	1	0	-1

Showing the fourth animal (horse) to the perceptron:
$x_1(4)w_1(4) + x_2(4)w_2(4) + x_3(4)w_3(4) + x_4(4)w_4(4) = 0*-1 + 0*1 + 0*0 + 1*-1 = -1 \leq 0$
So, $g(4) = 0$
$e(4) = y(4) – g(4) = 0 – 0 = 0$
The weights don't change.

Step 5:

w_1	w_2	w_3	w_4
-1	1	0	-1

Showing the last animal (fish) to the perceptron:
$x_1(5)w_1(5) + x_2(5)w_2(5) + x_3(5)w_3(5) + x_4(4)w_4(4) = 0*-1 + 1*1 + 0*0 + 1*-1 = 0 \leq 0$
So, $g(5) = 0$
$e(5) = y – g(5) = 0 – 0 = 0$
The weights don't change.
After showing the last sample we say that one learning cycle (epoch) is finished.

Step 6:

w_1	w_2	w_3	w_4
-1	1	0	-1

Showing the first animal (eagle) to the perceptron for the second time (second epoch):
$x_1(6)w_1(6) + x_2(6)w_2(6) + x_3(6)w_3(6) + x_4(6)w_4(6) = 1*-1 + 1*1 + 1*0 + 1*-1 = -1 \leq 0$
So, $g(6) = 0$
$e(6) = y(6) – g(6) = 1 – 0 = 1$
In this case the weights must change.
$\Delta w_1(5) = e(6)\, x_1 = 1*1 = 1$
$\Delta w_2(6) = e(6)\, x_2 = 1*1 = 1$
$\Delta w_3(6) = e(6)\, x_3 = 1*1 = 1$
$\Delta w_4(6) = e(6)\, x_4 = 1*1 = 1$
So, according to the delta rule, new weights are:
$w_1(7) = w_1(6) + \Delta w_1(6) = (-1) + 1 = 0$
$w_2(7) = w_2(6) + \Delta w_2(6) = 1 + 1 = 2$
$w_3(7) = w_3(6) + \Delta w_3(6) = 0 + 1 = 1$
$w_4(7) = w_4(6) + \Delta w_4(6) = (-1) + 1 = 0$

The learning process will continue in this manner until the weights are good for all the samples.
When the error is null for the whole training set, the learning process is over.
After four epochs, the weights that settle the whole learning set are:

w_1	w_2	w_3	w_4
0	2	1	-2